

A Note on the Chiral Anomaly in the AdS/CFT Correspondence and $1/N^2$ Correction

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Abstract

According to the AdS/CFT correspondence, the $d = 4$, $\mathcal{N} = 4$ $SU(N)$ SYM is dual to the Type IIB string theory compactified on $AdS_5 \times S^5$. A mechanism was proposed previously that the chiral anomaly of the gauge theory is accounted for to the leading order in N by the Chern-Simons action in the AdS_5 SUGRA. In this paper, we consider the SUGRA/string action at one loop and determine the quantum corrections to the Chern-Simons action. While gluon loops do not modify the coefficient of the Chern-Simons action, spinor loops shift the coefficient by an integer. We find that for finite N , the quantum corrections from the complete tower of Kaluza-Klein states reproduce exactly the desired shift $N^2 \rightarrow N^2 - 1$ of the Chern-Simons coefficient, suggesting that this coefficient does not receive corrections from the other states of the string theory. We discuss why this is plausible.

1 Introduction

According to the AdS/CFT correspondence [1, 2, 3, 4], the $\mathcal{N} = 4$ $SU(N)$ supersymmetric gauge theory considered in the 't Hooft limit with $\lambda \equiv g_{YM}^2 N$ fixed is dual to the IIB string theory compactified on $AdS_5 \times S^5$. The parameters of the two theories are identified as $g_{YM}^2 = g_s$, $\lambda = (R/l_s)^4$ and hence $1/N = g_s(l_s/R)^4$. In general, a gauge theory diagram with genus g comes with a power of N^{2-2g} and with some powers of λ . In the string theory, correlation functions for a given genus g come with a power of g_s^{2-2g} and have a coefficient given by a power series expansion in $\alpha'/R^2 = \lambda^{-1/2}$. At a given order of N or g_s , the functions of λ have different expansions (powers of λ versus powers of $\lambda^{-1/2}$) in the two theories, although according to the AdS/CFT proposal, they are supposed to represent the same function. It is clear that comparison between the two theories is possible only for quantities whose λ dependences can be computed exactly or that are independent of λ . Since the anomaly is independent of λ , it is a perfect candidate for this purpose.

In the $\mathcal{N} = 4$ SYM, we have two kinds of anomaly, the trace anomaly and the $SU(4)_R$ chiral anomaly. Both have been checked [3, 5, 6] and found to match with SUGRA calculations to leading order at large N . Subleading orders in N for other gauge systems have also been discussed [7, 8]. However, the $1/N^2$ corrections for the original $\mathcal{N} = 4$ case have never been tested. In this paper, we will be interested in the $1/N^2$ corrections to the chiral anomaly. To leading order in N , the chiral anomaly has been elegantly accounted for [3, 5] by the Chern-Simons action in the AdS_5 SUGRA. The coefficient k (see (9) below) determines the magnitude of the chiral anomaly in field theory and is given by $k = N^2$ to the leading order in N . For finite N , the chiral anomaly of the $SU(N)$ gauge theory is proportional to $N^2 - 1$ and there is a mismatch of -1. Our goal is to determine the $1/N^2$ correction to this coefficient k and reproduce this “-1” correction.

According to the proposal [1, 2, 3], quantities of order $1/N^2$ corresponds to a 1-loop string calculation. Although a classical string action on $AdS_5 \times S^5$ with RR fields background is available [9], the quantization of it is notoriously difficult and we still don't have reliable means to compute its quantum corrections. This is partially reflected by the fact that we still don't know the complete spectrum of states of the string theory on $AdS_5 \times S^5$. The only explicitly known states are the full towers of KK states [10] coming from the compactification of the 10 dimensional IIB SUGRA multiplet. It is thus natural to first examine the quantum corrections to the Chern-Simons coefficient coming from all of these states. In fact, we find precisely the expected correction of -1. This correction of -1 is entirely due to the quantum treatment of the doubleton multiplet that is gauged away. This is very satisfactory since the doubleton is known to be dual to the decoupled $U(1)$ factor of the SYM theory which is $SU(N)$ rather than $U(N)$. Given that this already provide the desired shift $N^2 \rightarrow N^2 - 1$, we expect that there is no contribution from other string states at all. We also note that a consistent truncation to the states of the 5

dimensional supergravity multiplet alone is not sufficient.

We will first review in sec. 2 the arguments [2, 5] for matching the chiral anomaly with the Chern-Simons term in the five dimensional SUGRA action to leading order in N . Then in sec. 3 we determine the one loop corrections to the coefficient of the Chern-Simons action from the Kaluza-Klein towers. We finally discuss why it is plausible that other string states don't modify the Chern-Simons coefficient.

2 Chiral Anomalies in the $\mathcal{N} = 4$ SYM

The $\mathcal{N} = 4$ SYM in 4 dimensions has a R-symmetry group of $SU(4)_R$. The matter fields are in nontrivial representation of it, with the scalars X^i transforming in the **6** and the four complex Weyl fermions λ in the fundamental representation of $SU(4)_R$ with the chirality part (0,1/2) in **4** and (1/2,0) in **4*** (see for example [11]. Our convention here together with the convention we adopt in (17) are equivalent to those these authors used.) It is convenient to use the compact notation of differential forms [12, 13]. The correctly normalized anomaly is derived from the descent equation

$$\frac{i^{n+2}}{(2\pi)^n(n+1)!} \text{Tr} F^{n+1} = d\omega_{2n+1}(A), \quad \delta_v \omega_{2n+1} = d\omega_{2n}^1(v, A), \quad (1)$$

where

$$F = dA + A^2, \quad \delta_v A = dv + [A, v] \quad (2)$$

and $v = v^a T^a$, $A = A^a T^a$. v^a are commuting gauge parameters and T^a 's are anti-Hermitian and are in the fundamental representation. Explicitly for $n = 2$,

$$\omega_4^1(v, A) = \frac{1}{24\pi^2} \text{Tr} [vd(AdA + \frac{1}{2}A^3)], \quad (3)$$

$$\omega_5(A) = \frac{1}{24\pi^2} \text{Tr} [A(dA)^2 + \frac{3}{2}A^3dA + \frac{3}{5}A^5]. \quad (4)$$

In this notation, the R-symmetry anomaly is

$$v \cdot (D_i J^i) = -(N^2 - 1) \int_{S^4} \omega_4^1(v, A), \quad (5)$$

where we used the convention for Einstein summation:

$$v \cdot F = \int d^4x v^a(x) F^a(x) \quad (6)$$

for any $v^a(x)$ and $F^a(x)$. The $N^2 - 1$ factor is due to the fact that λ is in the adjoint of $SU(N)$.

For T^a in a general representation \mathbf{R} of the group, the corresponding quantities with the trace taken in \mathbf{R} are

$$\omega_{2n}^1{}^{\mathbf{R}} = A(\mathbf{R})\omega_{2n}^1, \quad \omega_{2n+1}^{\mathbf{R}} = A(\mathbf{R})\omega_{2n+1}, \quad (7)$$

where $A(\mathbf{R})$ is the anomaly coefficient defined by the ratio of the d -symbol taken in the representation \mathbf{R} and the fundamental representation. In general $2n$ or $2n+1$ dimensions, since the d -symbol is given by a symmetrized trace of $n+1$ Lie algebra generators, it is easy to show that the complex conjugate representation \mathbf{R}^* has an anomaly coefficient

$$A(\mathbf{R}^*) = (-1)^{n+1}A(\mathbf{R}). \quad (8)$$

According to the AdS/CFT proposal, one should be able to see this anomaly from the dual point of view of string theory. To leading order in N , one looks at the IIB SUGRA compactified on $AdS_5 \times S^5$. The tree level SUGRA action contains the term [14, 15, 3, 5]

$$S_{cl}[A] = \frac{1}{4g_{SG}^2} \int d^5x \sqrt{g} F_{\mu\nu}^a F^{\mu\nu a} + k \int_{AdS_5} \omega_5. \quad (9)$$

We note that in terms of components, $k \int \omega_5 = \frac{ik}{96\pi^2} \int d^5x d^{abc} \epsilon^{\mu\nu\lambda\rho\sigma} A_\mu^a \partial_\nu A_\lambda^b \partial_\rho A_\sigma^c + \dots$, if one uses Hermitian generators and the definition of d -symbol in [5]. From the dual gauge theory point of view, since the current J^a is coupled to the $SU(4)$ gauge fields of the bulk, using the AdS/CFT proposal one can determine the coefficient g_{SG}^2 and k from the 2-point and 3-point correlators of J^a [5] of the gauge theory. The normalization to leading order in N

$$g_{SG}^2 = \frac{16\pi^2}{N^2}, \quad k = N^2, \quad (10)$$

has been determined in [5]. The ratio of the coefficient g_{SG}^2 and k also agrees with what one gets from supersymmetry [14, 15]. One may also determine the values of g_{SG}^2 and k from a dimensional reduction of the 10 dimensional IIB SUGRA. This requires the knowledge of the N dependence of R . According to the proposal [1, 2, 3], the radius R of S^5 is determined by $R^4/\alpha'^2 = g_s^2 N$. Using this, it is easy to determine the normalization of the gauge kinetic energy term and one indeed finds $g_{SG}^2 = 16\pi^2/N^2$ and hence $k = N^2$ using SUSY.

In usual consideration of SUGRA on AdS , one considers gauge configurations A^a which vanish at the boundary and so the Chern-Simons term is gauge invariant. For the consideration of AdS/CFT correspondence, the boundary value of the A^a is nonvanishing and coupled to the R-currents J^a . Under a gauge variation $\delta_v A$, the variation of the Chern-Simons term is a boundary term

$$\delta_v S_{cl} = k \int_{S^4} \omega_4^1. \quad (11)$$

Now by the conjecture [1, 2, 3],

$$S_{cl}[A_\mu^a(x, x^5)] = \Gamma[A_i^a(x)], \quad (12)$$

where Γ is the generating functional for current correlators in the boundary theory, and hence

$$\delta_v S_{cl} = \delta_v \Gamma = -v \cdot (D_i J^i). \quad (13)$$

From this and (11), one can read off the anomaly. It is

$$v \cdot (D_i J^i) = -N^2 \int_{S^4} \omega_4^1(v, A), \quad (14)$$

which agrees with the gauge theory computation (5) to leading order in N .

3 Induced Chern-Simons

As we have seen in the previous section, the IIB tree level SUGRA contains a Chern-Simons term which can account for the chiral anomaly of the gauge theory to leading order in N . But there is also a mismatch of “-1” which is of order $1/N^2$. In this section, we will examine the $1/N$ corrections to the Chern-Simons action on the string theory side. From the point of view of the IIB string theory, an expansion in $1/N$ is a quantum expansion beyond tree level. In particular the $1/N^2$ correction to the chiral anomaly corresponds to a 1-loop computation in IIB string theory. We will first examine the corrections coming from the Kaluza-Klein states. Based on the origin of the Chern-Simons action in AdS_5 supergravity, we will argue in the discussion section that the other string states are not likely to modify the Chern-Simons coefficient.

Fermionic contributions

It is well known that chiral fermions in even dimensions can give rise to an anomaly. Although there is no chirality in odd dimensions, there is a similar phenomenon for fermions in odd dimensions. Consider a Dirac fermion ψ in odd dimensions (flat) minimally coupled to (external or gauge) vector bosons A_μ of a group G . At the quantum level, a regularization needs to be introduced to make sense of the theory and one cannot preserve both the gauge symmetry (small and large) and the parity at the same time [16, 17]. If one chooses to preserve the gauge symmetry by doing a Pauli-Villars regularization, then there will be an induced Chern-Simons term generated at one loop. The result is independent of the fermion mass [17, 18]. In our notation, the induced Chern-Simons term is

$$\Delta\Gamma = \pm \frac{1}{2} \int \omega_{2n+1}^{\mathbf{R}} = \pm \frac{1}{2} A(\mathbf{R}) \int \omega_{2n+1}, \quad (15)$$

where \mathbf{R} is the representation of the Dirac fermion. The \pm sign depends on the regularization and can often be fixed within a specific context.

This result was originally [16, 17] obtained for fermions coupled to gauge fields in a flat spacetime and has been extended to full generality for arbitrary curved backgrounds and any odd dimensions [18]. The induced parity violating terms are given (up to a normalization factor) by the secondary characteristic class $Q(A, \omega)$ satisfying

$$dQ(A, \omega) = \hat{A}(R)ch(F)|_{2n+2}, \quad (16)$$

where ω is the gravitational connection. Going one step down the descent relation, one gets the chiral anomaly of a Dirac operator defined on a curved manifold. Since $\hat{A}(R) = 1 + o(R^2)$ and $Tr F = 0$ for SU , in five dimensions there is only the gauge Chern-Simons and the gravitational Chern-Simons terms from (16) and there is no mixing term. It is clear that the pure gauge Chern-Simons piece takes the same expression (15) as in the flat case. This can indeed be expected from the beginning as the gauge Chern-Simons form is independent of the metric. It is also clear that there is no induced gravitational Chern-Simons term as it would be related to a gravitational anomaly in four dimensions, and gravitational anomalies exist only in $4k + 2$ dimensions.

Now we need the particle spectrum of the type IIB string theory on $AdS_5 \times S^5$. The only explicitly known states are the KK states coming from the compactification of the 10 dimensional IIB SUGRA multiplet [10]. So we will examine them first. Particles in AdS_5 are classified by their unitary irreducible representation of $SO(2, 4)$. Since $SO(2, 4)$ has the maximal compact subgroup $SO(2) \times SU(2) \times SU(2)$, irreducible representations are labelled by the quantum numbers (E_0, J_1, J_2) . The complete KK spectrum of the IIB SUGRA on $AdS_5 \times S^5$ was obtained in [10] together with information on the masses (in units of $1/R$)¹ and the representation content under $SU(4)_R$. We reproduce these results in the following table where we also give the $SU(2) \times SU(2)$ content [19] for the fermionic towers.

	$SU(2) \times SU(2)$	masses		$SU(4)_R$	
ψ_μ	$(1, 1/2)$	$k + 3/2$	$k \geq 0$	4 , 20 , ...	←
	$(1, 1/2)$	$-(k + 7/2)$	$k \geq 0$	4* , 20* , ...	
λ	$(1/2, 0)$	$-(k - 1/2)$	$k \geq 1$	20* , ...	←
	$(1/2, 0)$	$k + 11/2$	$k \geq 0$	4 , 20 , ...	
λ'	$(1/2, 0)$	$-(k + 3/2)$	$k \geq 0$	4* , 20* , ...	←
	$(1/2, 0)$	$k + 7/2$	$k \geq 0$	4 , 20 , ...	
λ''	$(1/2, 0)$	$-(k + 9/2)$	$k \geq 0$	36 , 140 , ...	
	$(1/2, 0)$	$k + 5/2$	$k \geq 0$	36* , 140* , ...	

(17)

The fermions in this table are symplectic Majorana spinors. For simplicity, we have only listed half of the field content. The other half (“mirror”) consists of fields with conjugate $SU(4)_R$ content and with the $SU(2) \times SU(2)$ quantum numbers exchanged [19]. The

¹There seems to be a misprint for the masses of the spin 3/2 in the table of [10].

first member of the rows marked with an \leftarrow together form the fermionic sector of the $\mathcal{N} = 8$ supergravity multiplet: 8 gravitini and 48 spin $1/2$. We have chosen to list the “left handed” spinors here. The “chirality” refers to the anti-de Sitter group $SU(2, 2)$. Since the Chern-Simons term in odd dimensions is related to the chiral anomaly in one lower dimension [18], the “chirality” thus allows us to fix the \pm sign of the induced Chern-Simons term. In particular, the “right-handed” (“left-handed”) spinor generates a Chern-Simons term with $+$ ($-$) sign in (15). We should also remember (8) that the conjugate representation \mathbf{R}^* has an opposite anomaly coefficient in five dimensions. To determine the net induced Chern-Simons terms from all these states, we should also sum over the “mirror”. Notice that the “right-handed” sector contributes exactly the same as the “left-handed” sector because relative to the “left” sector, they get one minus sign from the “chirality” and another one from $A(\mathbf{R}^*)$. Notice also that the induced Chern-Simons term from a symplectic Majorana spinor is half that of a Dirac spinor because it contains half as many degrees of freedom. Therefore effectively we can just sum over the spectrum in the “left handed” sector above using the formula

$$\Delta\Gamma = -\frac{1}{2}A(\mathbf{R}) \int_{AdS_5} \omega_5 \quad (18)$$

as if they are Dirac spinors.

It is clear that the towers of $\psi_\mu, \lambda', \lambda''$ do not generate a net induced Chern-Simons term as the states in these towers all comes in pairs with their $SU(4)$ content conjugate of each other. However, as pointed out in [10], there is a missing $k = 0$ state ($\mathbf{4}^*$) in the first tower of λ . There are similar “missing states” in the bosonic towers. Together they are identified with the doubleton multiplet of $SU(2, 2|4)$ which consists of a gauge potential, six scalars and four complex spinors. These are nonpropagating modes in the bulk of AdS_5 and can be gauged away completely [10, 20], which is the reason why they don’t appear in the physical spectrum. These modes are exactly dual to the $U(1)$ factor of the $U(N)$ SYM living on the boundary [4]. Since from the SYM point of view, the -1 correction to the chiral anomaly is due the decoupling of this $U(1)$ to give $SU(N)$, we expect that from the dual point of view of the AdS_5 string theory, the -1 correction would also find an explanation entirely in terms of these gauge modes. Indeed, since there is an additional $k = 0$ state (“left-handed” and in $\mathbf{4}$) in the tower of λ which is not balanced out, there will be a Chern-Simons term of -1/2 (in units of $\int \omega_5$) coming from this state. This is only half of the desired result. However this is not the complete story.

Let us recall the doubleton multiplet is absent because it has been *gauged away* [10] by imposing the gravitino gauge fixing condition (see also [20] for the gauging in the case of $AdS_7 \times S^4$ case). Hence to properly quantize the system, one has to introduce the corresponding Faddeev-Popov ghosts. These ghosts will give another contribution of $-1/2$ to the Chern-Simons coefficient. Indeed the ghost multiplet contains bosonic spinors in exactly the same $SU(4)_R$ representation as the spinors of the doubleton multiplet, i.e. in $\mathbf{4}^*$. Natively one would expect a contribution opposite to the one of the unbalanced

$k = 0$ state (in **4**) of the λ tower. However, since the ghosts have opposite statistics, they actually give the same contribution, i.e. another $-1/2$. So altogether we get a total induced Chern-Simons term of -1 ,

$$\Delta\Gamma = - \int_{AdS_5} \omega_5, \quad (19)$$

which is exactly the desired result. Notice that the induced Chern-Simons (coming with a constant integer coefficient) is independent of the radius R and this is consistent with the AdS/CFT proposal since the anomaly and its corrections are independent of λ .

Bosonic contributions

There is another interesting effect related to the Chern-Simons action. It is known that in three dimensions, the gluons at one loop can modify the coefficient of the Chern-Simons action by an integer shift. However, the precise modification depends on the regularization scheme adopted ². In five dimensions, it is easy to convince oneself that there is no induced Chern-Simons Lagrangian coming from the gluon loops. The reason is simple. Suppose one adopt some regularization scheme to compute the gluon loops that may generate an induced Chern-Simons term. Since the gluons are in the adjoint representation of the gauge group, the diagrams are proportional to the d -symbol of the adjoint representation, or $A(\mathbf{adj})$. For three dimensions, this is proportional to the quadratic Casimir, but it is zero in five dimensions. In general, (8) says that $A(\mathbf{R}^*) = A(\mathbf{R})$ for $4k+3$ dimensions and $A(\mathbf{R}^*) = -A(\mathbf{R})$ for $4k+1$ dimensions. Hence $A(\mathbf{adj}) = 0$ for the present case. That the gauge bosons do not modify the Chern-Simons coefficient was already noted by Witten [2]. Moreover, since the other members (**64**, **175**, \dots) of the spin 1 towers [10] are all in real representations, they also don't modify the Chern-Simons action.

We thus see that while gauge boson loops do not modify the magnitude of the Chern-Simons action, the spinor loops do. Due to the intimate relation with the chiral anomaly [18], one expects that there is a nonrenormalization theorem (counterpart of the Bardeen nonrenormalization theorem for the chiral anomaly) that protects the Chern-Simons action from further corrections beyond one loop.

Combining the above, we find that at finite N , the coefficient k is shifted by

$$k \rightarrow k - 1 \quad \text{or} \quad N^2 \rightarrow N^2 - 1 \quad (20)$$

due to the quantum effects of the full set of Kaluza-Klein states.

4 Discussion

We have seen that the finite N dependence of the $SU(4)_R$ chiral anomaly is exactly accounted for by the quantum effects of the full set of KK states. A consistent truncation

²We thanks R. Stora for a useful discussion about the issues of regularization.

to the 5 dimensional supergravity states does not do the job and has no reason that it should. In fact, the total induced Chern-Simons term has its contribution coming from the $k = 0$ state of the λ tower and from the ghost spinors to the doubleton multiplet, both have nothing to do with the five dimensional supergravity multiplet. Thus the -1 shift at finite N of the chiral anomaly find a direct explanation entirely in terms of the quantum effects of the doubleton multiplet in AdS_5 , which is known to be dual to the decoupled $U(1)$ factor of the gauge theory. The whole picture is consistent.

For the AdS/CFT proposal to work at finite N , the other string states shouldn't contribute any further corrections to the Chern-Simons coefficient. While we don't have a proof of this statement, we would like to argue that this is plausible.

In [21] the Kaluza-Klein origin of the Chern-Simons action in gauged supergravity theory was explored. In particular, it was shown that for the case of $AdS_7 \times S^4$ compactification of the 11 dimensional SUGRA, the Chern-Simons term comes from the 11-dimensional Chern-Simons coupling $\int C \wedge G \wedge G$ upon compactification. The origin of the Chern-Simons term in $AdS_5 \times S^5$ SUGRA was also discussed and it is expected to arise similarly from the compactification of the 10 dimensional IIB SUGRA. However, there are some technical subtleties, the major one being the lack of a covariant action of the IIB SUGRA. It would be very interesting to work it out explicitly, presumably the formalism of [22] will be useful.

Given that the Chern-Simons term has its origin in the 10 dimensional SUGRA multiplet, it is natural to expect that the quantum corrections to the Chern-Simons action are also confined to the states related to the Kaluza-Klein compactification and we conjecture that this is indeed the case.

Since the trace anomaly and the R-symmetry anomaly are in the same SUSY multiplet, a similar finite N correction of -1 should also appear in the trace anomaly. We expect that again the correction will have an explanation entirely in terms of the quantum effects related to the doubleton multiplet. It would be interesting to verify this explicitly. This will provide a cross check of our conjecture here.

Loop effects in AdS_5 supergravity are definitely highly nontrivial to compute. In fact, tree level calculations already call for the invention of many ingenious techniques and methods. Were it not for the topological character of the Chern-Simons action, one will not be able to determine exactly the parity violating term induced at one loop. For other processes involving loops, in addition to the difficulty of evaluating complicated momentum integrals, one also has a lot more states propagating since a truncation of states to the 5 dimensional supergravity multiplet is not enough. It would be interesting to find other quantities that can be calculated at one or more loops in order to provide other tests of the AdS/CFT proposal at finite N .

Since Chern-Simons action exists generally in gauged supergravity on AdS_{p+1} space, they all have interpretation as chiral anomaly in the dual CFT. For example, in the $AdS_7 \times S^4$

case discussed in [21], the Chern-Simons action has a coefficient of N^3 and the $1/N^2$ correction would be of order N . We suspect that this correction would again find an explanation in terms of the doubleton multiplet, which now consists of a tensor gauge field, four spinors and five scalars.

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